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A Review on Decomposition-Based Multi-Objectives Evolutionary Algorithms

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ABSTRACT

The real-world optimization problems always have many conflicting objectives that need to be optimized at once. A comprehensive number of research studies try to solve such kind of problems. Multi-Objectives Evolutionary-Algorithms using Decomposition (MOEA/D) is one of the most powerful algorithms that solves both multi and many objective optimization problems. The basic idea of such algorithms is to convert the complex Multi-Objective Optimization Problem (MO-OP) into a set of uniobjective subproblems. This conversion is performed with the help of the information acquired from the neighborhood of each subproblem. The algorithm could efficiently solve the tradeoffs between both diversity of the proposed solutions and the convergence of the algorithm. Due to the simplicity and the efficiency of the algorithm, different research studies investigated the improvement and adaptation of MOEA/D. In this paper, a review of the different decomposition-based algorithms is proposed. The research studies covered in this paper is categorized into four groups: weight vector generation, scalarization and aggregation strategies, the MOEA/D variants and the MOEA/D real-world applications.

1. Introduction

Finding solutions that optimally fit the objectives for a certain problem; maximization or minimization is called an optimization problem. Such kinds of problems always have one or more objectives to satisfy. In the actual world, problems that involve conflicting goals or objectives is a major challenge. That's because there hardly exists a single solution that satisfies all the objectives at once.

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In this case, designers must choose between competing objectives as selecting the best value of an objective will be at the expense of other objectives. These problems are referred to as Multi-Objective Optimization Problems (MO-OP).

There is a large number of real-world applications that can be recognized as MO-OP, such as distributed network-configuration [1], network-routing, real-time scheduling, financial applications [2] , medical and health care decision-making [3], and drugs molecular-structure [4]. A MO-OP can be represented as shown by the following equation [5], [6]:

$$\text{Maximize } F(x) = (f_1\,(x)\,, \dots \dots f_m(x))^T \qquad (1)$$
$$\text{subject to } \quad x \in \Omega$$

where: m refers to the number of objectives, Ω the variable or decision-space, and F: $\Omega \to R^m$ is the objective-space [6].

The attainable objective-set is defined as $\{F(x)|\, x \in \Omega\}$. Due to the contradictions between the different objectives, the final solution for MO-OP is the set of the whole the non-dominated points [6], [7].

If both $u, v \in R^m$, It can be said that, u dominates v if $u_i \geq v_i$ for any $i \in \{1, \dots \dots, m\}$ and $u_j > v_j$ for at least a single index $j \in \{1, \dots \dots., m\}$ [6].

A point $x^* \in \Omega$ can be thought of as Pareto-optimal to Eq. (1) if it is not dominated by any other point x , S.T $x \in \Omega$. In this case, $F(x^*)$ is considered as optimal objective-vector. Any Pareto-optimal point's refinement of any of the objectives always causes regression into at least another one. The set containing all these points is referred to as Pareto-optimal-Set (PS), whereas the set containing the entire Pareto-optimal objective vectors is referred to as the Pareto-Front (PF) [7] [8].

The Multi-Objective Optimization (MOO) algorithms are Evolutionary-Algorithms (MOEAs) used to find the best obtainable solutions such that the solutions evolve or improve over a set of iterations. MOEAs are classified into two types: Pareto-Dominance-based (MOEA-PD) and Decomposition-based algorithms (MOEA/D).

Pareto-Dominance-based algorithms (MOEA-PD) generally provide sufficient performance with small objective spaces. On the other hand, they become unable to scale well with larger objective spaces and they lie into a selection pressure problem. Amongst the MOEA-PD algorithms are the Strength Pareto-Evolutionary-Algorithm (SPEA) [9], and Non-dominated-Sorting Genetic-Algorithm (NSGA2) [10].

MO-OP with high objective spaces where the objectives ≥ 3 are called many-objective-optimization problems. As mentioned, MOEA-PD fails to deal with such problems, where in this case, as the number of objectives increase none of the candidates can be dominant over one another.

Zhang & Li [11] found a solution for such kinds of problems by proposing a different type of algorithms that uses decomposition MOEA/D.

Transferring the MO-OP into a concurrent collection of uniobjective sub-problems is the basic idea behind MOEA/D algorithms. The simplification mechanism used by MOEA/D does not only reduce the complexity of the problem at hand, but also resolves the dominance-resistance problems of the MOEA-PD algorithms.

The decomposition for such algorithms is performed by aggregation of the weighted objectives such that each sub-problem has a different generated weight-vector. The weight values as well as the aggregation method used are the factors that affect the algorithm's performance. Due to the good performance shown by MOEA/D, different researchers have investigated the improvements of these factors as well as applying the decomposition approach to different real-time application areas.

As shown in Figure 1, the MOEA/D research areas lie into four groups: scalarizing-function adaptaion, weight vectors generation mechanisms, newly implemented MOEA/D versions, and MOEA/D different real-world application areas.

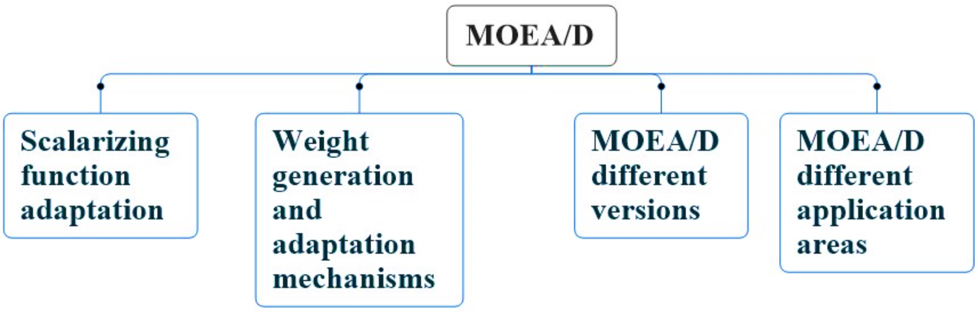

Figure 1. MOEA/D research categories

The following sections of the paper are organized as follows; Section 2 shows the preliminaries of the original MOEA/D algorithm. In section 3, the scalarizing functions adaptation mechanisms are proposed, whereas in section 4, the weight vectors generation strategies are presented. Section 5 shows the different versions of the MOEA/D algorithms. In section 6, the different real-world application areas are proposed. Finally, section 7 shows the research conclusions.

2. The preliminaries of the original MOEA/D

The MOEA-PD handle the MO-OP as a whole without decomposing the problem [5]. In MOEA-PD, the solutions' quality is determined based on the non-domination sorting principle. In this case, the solutions are ranked based on their PD. The non-dominated set of solutions are given the best rank.

The highest ranked solutions are favored throughout the selection or the update process as they lead the search towards convergence. To maintain diversity of solutions, the MOEA-PD algorithms apply diversity-promotion measures, such as weighted and crowding distance measures to achieve a balanced distribution of PF [12].

As previously mentioned, The MOEA-PD algorithms can efficiently approximate the Pareto Front (PF) for problems with low number of objectives. Nevertheless, the performance is significantly affected as a result of the increasing number of objectives. In such a case all the solutions will be nondominated to each other [11], [6]. The other problem with MOEA-PD is that it suffers from high computational-complexity.

For most MO-OP, covering the whole PF is very time consuming as they always have an infinite number of Pareto Optimal (PO) vectors. So, Zhang & Li [11] proposed a newly implemented MOEA using Decomposition (MOEA/D). The main idea of decomposition-based MOO techniques is to break down the MO-OP into a group of single objective sub problems.

MOEA/D is one of the most promising techniques for dealing with multi- and many-objective optimization problems (i.e., problems with more than 3 objectives) because each sub-problem is optimized concurrently and collaboratively using information from its neighboring sub-problems. As a result, the complexity of MOEA/D is reduced [11].

The first step to solve the MO-OP using MOEA/D is to decompose or split the problem defined by Eq. (1) into several scalar sub problems and to work on these sub problems concurrently. According to [11], there are many decomposition techniques or SFs; the Weighted-Sum (WS), the Penalty Boundary-Intersection (PBI), and the Weighted-Tchebycheff (W-Tch) technique.

The W-Tch will only be considered in this section as it is considered the most effective one. The MO-OP of Eq. (1) can be handled as a group of N scalar sub problems where, the objective function of the j^{th} sub problem using the W-Tch Scalarization is given as stated in [11] as:

$$\text{Minimize} \quad g^{w\_tch}(x \mid \lambda^j, z^*) = \max_{1 \le i \le N} \{ w_i^j |f_i(x) - z_i^*| \} \qquad (2)$$
$$\text{subject to } x \in \Omega$$

Where $z = (z_1^*, z_2^*, \dots \dots z_N^*)^T$ is the reference-point, $S.T\ z_i^* = \max\{f_i(x) | x \in \Omega\}$ for $i = 1 \to N$. For each Pareto-optimal point (i.e., non-dominated solution) x^*, there exists a weight vector $w = (w_1, \dots., w_N)^T$, such that $w_i^j \ge 0, and\ \sum_{i=1}^{N} w_i = 1$ for all i from 1 to N objectives and for all j from 1 to m sub-problems. In this case, all the non-dominated solutions found for Eq. (2) are considered as Pareto-optimal to Eq. (1).

By changing the weight vector, various Pareto-optimal solutions could be achieved. So that, selecting the appropriate weight vectors is one of the factors that affect the solution quality. For each weight vector w^i there is a neighborhood which is a set of the T closest weight vectors in $\{w^1, \dots., w^m\}$.

Hence, the neighbourhood of the ith sub problem contains every sub problem that has a weight vector at distance $\le T$ from w^i. Figure 2 shows the detailed decomposition-based algorithm proposed by Zhang et al. [11].

The original MOEA/D framework

Inputs:

- Multi-Objective Optimization (MOO) problem.
- m : The number of sub problems (the population size).
- W: A set of m evenly sampled weight vectors $\{w_1, \ldots, w_m\}$.
- T : The neighborhood size.
- A stopping criterion

Steps:

1. Initialization:
   - Generate the initial population $x^1 \to x^m$ at random such that m is the population size where, x^i is the current solution to the ith sub problem.
   - Initialize the reference point z as mentioned in Eq.(2).
   - Calculate the Euclidean distances for each couple of weight vectors to determine the neighbors for each vector (the set of its T closest weight vectors).
   - For each $i = 1 \to m$, set $B(i) = \{i_1, \cdots, i_T\}$, such that $w^{i_1}, \cdots, w^{i_T}$ are the T closest weight vector to w^i.
   - For each $i = 1 \to m$:Evaluate the fitness value $F(x^i)$.
2. Update:

   For $i = 1 \to m, do$
   - Reproduction: Select at random two indexes k, l from $B(i)$ then, by using the genetic operators (i.e., crossover and mutation) generate a new solution from x^k and x^i.
   - Repair: Apply a problem specific improvement heuristic on y to generate y'.
   - Update z: For each $j = 1 \to N$, If $z_j < f_j(y')$ then set $z_j = f_j(y')$.
   - Neighboring solutions update: For each index $j \in B(i)$,

     $if\ g^{te}(y'|w^j, z) \leq g^{te}(x^j|w^j, z),\ then\ set\ x^j = y'$ and $FV^j = F(y')$.
3. If stopping criteria is met, then stop. Else go back to step 2.

Figure 2. The original MOEA/D algorithm steps

3. Scalarization function adaptation mechanisms

The Scalarization Function (SF) or aggregation function has a crucial role in MOEA/D algorithms. It is responsible for the transformation of the MO-OP into a set of scalar subproblems [13], [14]. In addition, the selection of the appropriate scalarization function affects to a high degree the search ability of the used algorithm.

The research studies concerning the scalarization function adaptations lie into two categories: combining two or more scalarizing functions together into a single algorithm or generation of new scalarizing functions that were not previously used in the literature.

In [15] , the simultaneous application of the W-Tch and the WS in a single algorithm had been studied. There are two suggested implementation plans. First, there is the multi-grid system. Every SF in this approach has a full grid of equally distributed weight vectors, and each SF uses both of its equivalent grids at the same time. This design will double the size of the population and the actual number of neighbours, allowing the two grids to overlap. In the second plan, there is just one grid layout with several SFs. Every SF possesses an incomplete weight vector grid in which every function is apportioned to a weight vector in turn. The simplicity of the implementation as well as the ability to be expanded and applied to other SFs are the main advantages of the proposed schemes.

In [16], Ma et al. proposed a W-Tch decomposition with constrained Lp-norm. In this case, sub-problems are constructed using direction vectors rather than weight vectors. Whereas in [17] Pescador-Rojas & Coello proposed the simultaneous use of a number of different scalarization functions proposed as pools of functions. In order to produce uniformly-distributed solutions throughout the PF, they proposed combining SFs with similar target directions. The selection strategy chooses from a pool the SF that fits each sub population based on the improvement fitness rate that is computed for each sub problem at each generation.

Qi et al. [18] presented a novel scalarization function using a series of new reference points. These points are derived from a reference point specified by the decision maker in the preference model. Based on the developed scalarization function, they developed a user-preference-based algorithm, named R-MOEA/D.

Rodríguez & Coello [19] proposed new scalarization functions designed using Grammatical Evolution (GE). In this case the benchmark problems used are categorized based on their geometrical properties. Then, the used scalarization functions are applied to each problem independently. After that, the problems with the same properties are combined and the GE is used to generate new scalarization functions among the functions that were in use. In this case, the newly generated functions are only suitable to the types of problems they were designed for, and their performance reduces with the other types of problems.

Zheng & Wang [20] proposed a new Lp scalarization family based on the Global Replacement strategy(GR) called the GLp scalarization. The GLp based subproblem's direction vector is guaranteed to pass through its corresponding preference region, which gurantee that MOEA/D-GR can always avoid mismatches when using the GLp scalarization for any $p \geq 1$.

4. Weight vector generation and adaptation

One of the primary factors influencing the MOEA/D search process, as previously mentioned, is the weight assignment mechanism. Identical or non-well distributed weight vectors result in weak solutions that cannot cover the entire PF [11], [21].

There are two types of weight vector generation approaches: systematic and random. In systematic generation, the weights are created in a repeating pattern to guarantee evenly distributed vectors throughout the PF.

As the systematic weight distribution is only suitable for problems with regular or contineous PFs, it becomes unable to handle more complex problems i.e., problems with scattered, degenerated or irregular PFs. So, to remedy this problem many research studies have invistigated using adaptive weight generation mechanisms.

Jiang Siwei et al. [22] presented a Pareto-adaptive-weight-vector methodology called (paλ) which relies on Mixture-Uniform-Design (MUD). In this case, the weight vectors are modified based on the PF shape.

Another adaptive weight generation approach, known as MOEA/D-AWG, was presented in [23]. The results verified the efficiency of the proposed adaptively weight generation method as compared to MOEA/D alternatives with uniformly generated weight vectors. The suggested approach creates the weight vectors related to the geometrical characteristics of the PFs that are initially estimated by employing Gaussian process regression.

In [24], Junqueira et al. proposed an algorithm based on decomposition that adapts progressively its weight vectors during the evolution process. The algorithm is called Multi-objective Evolutionary Algorithm based on Decomposition with Local-Neighborhood Adaptation (MOEA/D-LNA).

Recently, Gu et al. [25] presented a new weight adaptive updating algorithm (called MOEA/D-AWS). Initially, the evolutionary matrix similarity subproblem is utilized to decide when to modify the weight vectors. In order to promote population variety, a subspace of weight vectors is created and used to partition the objective space. Finally, based on the size of the subspace, partial weights are given another chance to be chosen.

On the contrary to the systematic (uniform) generation, the random weight generation create weight vectors that are not necessarily similar which in turn provide a more throgh investigation to the search-space. However, the problem with randomly generated weights is that there is no gurantee to cover the whole PF [26]. So, some research studies have invistigated the use of both uniform and random weight generations in order to benefit from their advantages at once.

In [27] Li et al. proposed an improved version of MOEA/D to solve the problems with complex and irregular PFs. Two modifications have been taken into account. First, an external population is maintained during the search that is controlled by an adaptive-archiving strategy based on epsilon-dominance. On the other hand, some subproblems are optimized using random search strategies in case that they are not improved during the search process for a certain number of function evaluations. Otherwise, the other subproblems are optimized using systematic weight distributions.

Farias et al. [28] proposed a Uniformly-Randomly-Adaptive algorithm named as (MOEA/D-URAW). They used the same adaptation strategy presented in [26] combined with the uniform distribution for subproblems generation. In this case, the sub problems are generated based on the sparseness of the population.

In [29], the study proposed a new algorithm named MOEA/D-VW, which is derived from the original MOEA/D by adaptively altering the subproblems' weight vectors throughout the iteration. They construct weight vectors using the uniform random sampling approach first, and then they modify the weight vectors by changing the population's direction after each iteration. In order to reduce computational complexity and increase population competitiveness, they also offer a modified crossover operator.

Omran et al. [30] presented a new hybrid weight generation strategy merging both the uniform and random weight generations into a single algorithm. The subproblems are divided into two partitions, such that the first partition represents around 80% of the total number of subproblems are created using uniform weight generation in order to cover the PF. Whereas, the rest of the subproblems are creayed using random weight generation so as to get advantage of each. In this case the neighborhood for each subproblem will not be constant during the different iterations which improves the experience of each subproblem and enhance both the diversity and convergence of the algorithm.

5. MOEA/D different versions

Different variations of the original MOEA/D have been found in the literature. These algorithmic versions are divided into two groups. The first one is to apply the decomposition principle to other evolutionary algorithms. For example, MOEA/D-ACO [31], MO-GPSO/D [32], and MOEA/DD-CMA [33]. Whereas the other is to adapt the original MOEA/D to handle the more complex problems.

Ke et al. [31] suggested applying the decomposition approach to the Ant-Colony-Optimization named as MOEA/D-ACO. In this algorithm, they divide the number of ants by the number of subproblems, with each ant attempting to solve a single subproblem. Ants are divided into groups, and each group focuses on a certain area of the PF. Such that, each ant may have neighbors who belong to the same group or to a different one.

Martínez et al. [32] studied the application of Geometric-Particle-Swarm-Optimization (GPSO) [34]. The algorithm is designed to solve the discrete optimization problems. The algorithm was tested on many objectives problems i.e., problems with more than three objectives.

In [33], Castro et al. combined the Covariance-Matrix-Adaptation MOEA/D-CMA strategy using decomposition proposed by [35] with the MOEA using Decomposition-Dominance MOEA/DD proposed by [36]. The new algorithm is called MOEA/DD-CMA. This algorithm was compared with MOEA/D-CMA over some problems up to 15 objectives.

As previously mentioned, some algorithms proposed updates to the original MOEA/D in order to handle the complex irregular PFs. In [37], Jiang et al. used niching approach in conjunction with a Two Phase (TP) strategy called MOEA/D-TPN. In order to determine the form of the PF, the algorithm looks for regions where solutions are concentrated during the first phase. Depending on the outcomes of the first phase, the algorithm chooses the subproblem form to be employed in the second phase. By directing the mating process with parents in the areas with the least amount of population, the niching approach is proposed to prevent duplicate offsprings.

Xu et al. [38] suggested a hierarchical decomposition based MOEA called (MOEA/HD), which splits the subproblems into several layers/hierarchies and uses superior directing subproblems to modify the direction of search of the lower hierarchy subproblems.

Recently Chen et al. [39] introduced an innovative solution generation operator for MOEA/D. In this paper, they generate plausible candidate solutions by analyzing the movements of solutions from previous and current generations. The experimental results demonstrated that the suggested approach substantially accelerates the rate of convergence for MOEA/D.

6. MOEA/D applications

Due to the ability of the MOEA/D different algorithmic versions to solve hundreds of benchmark optimization problems, it was applied to many real-world optimization problems.

The most famous real-world applications found in the literature are:

- Engineering applications: Examples for the decomposition-based engineering applications are the sizing of a folded-cascode amplifier [40], reservoir flood-control operation [41], Electric-motors design [42], optimal power flow [43], hybrid energy systems [44] and antenna-design [45].
- Network applications: Finding the optimal mobile agent routes [46], multicast routing with network coding optimization problem [47], Community Detection in Large-Scale Complex Networks [48] and [49].
- Medical applications: Medical image segmentation [50], Cancer diagnosing, where [51] applied MOEA/D for a medical cancer gene expression for 35 datasets.
- Financial applications: Portfolio optimization [52], [53] and cryptocurrency algorithmic-trading optimization [30].
- Space and satellite applications: Space craft control-structure design [54], and aerospace applications [55], Satellite range scheduling [56], UAV 3-D path planning [57], and trajectory planning for parafoil UAVs, structural optimization for space trusses [58].

7. Conclusion

Decomposition is a simple yet efficient strategy that was previously used for traditional MO-OPs. Nevertheless, it was not applied to evolutionary based algorithms until Zhang and Li proposed their first decomposition-based algorithm MOEA/D.

The concept of MOEA/D is to simplify the MO-OP into a set of scalar subproblems depending on the acquired information through the neighboring subproblems. Such that the neighborhoods are determined based on the closest weight vectors that are assigned to each subproblem. Two main factors affect the performance of this type of algorithms: the weight vectors assignment strategies and the scalarization function used.

Due to the good performance shown by the MOEA/D algorithm, different research studies proposed variant updates to the original algorithms in different areas such as weight generation and adaptation, scalarization functions generation, proposing other versions of the original MOEA/D and applying the original MOEA/D and its variants to different other application areas. This paper proposed a review of the different variants of the MOEA/D found in the literature.

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